Why ancient cathedrals stand up

The structural design of masonry

How could the great architect-engineers of the past be sure that their buildings would stand up? What insights can we gain from their working practices? Professor Jacques Heyman explores the techniques they used and relates them to present-day structural design.

A Gothic cathedral was designed by a man who was (to use modern terms) both architect and engineer. The ‘master of the work’ had survived the long training of apprentice to journeyman to the career grade of master, and had been one of those outstanding masters who were put back to school again in the design office, before finally achieving control of a major work.

This educational path contrasts strongly with that of present-day Western European practice, which is based on the Renaissance concept of the ‘gentleman’ architect. In England the precursor is Inigo Jones, but the first real architect is Wren, a professor of astronomy who had never worked on a building site. Despite his lack of practical training, Wren established the first modern architectural practice, with
large unstable displacements, whether locally or overall.
An immediate difficulty arises when these ideas are applied to masonry. Ancient structures – a Greek temple, the Roman Pantheon – seem intuitively to be strong enough; many are still standing, and evidently the loading (self-weight, wind, earthquake) has been resisted robustly. Similarly, the engineer is unlikely to worry unduly about large working deflexions of the vault of a Gothic cathedral. Further, instability is usually encountered as a local phenomenon; a slender steel column must be designed not to buckle, whereas the masonry pier in a nave arcade is ‘stocky’. In short, conventional ideas must be abandoned when approaching the analysis of masonry.

The strength of stone
The masonry structure is indeed ‘strong’. Reference to nineteenth-century practice is illuminating. An indirect parameter was used to express the strength of the stone chosen for the design of great masonry arches – the height to which, theoretically, a column of the stone might be built before crushing at its base due to its own weight. For a medium sandstone, the height is 2 km; for granite, 10 km. The tallest Gothic cathedral, Beauvais, measures 48 m to the top of the stone vault; of course, the piers carry more than their own weight, and must support the vault, the timber roof, and wind forces – nevertheless, stresses are very low. In round figures, the crossing piers carrying a tower in a cathedral will be working at an average stress of less than one tenth of the crushing strength of the stone; main structural elements – flying buttresses, webs of masonry vaults – at one hundredth; and infill panels and the like at one thousandth of the potential of the material.

A vast masonry building is made by piling small stone blocks one on another – the resulting shape is given a label such as Byzantine, Romanesque or Gothic. If the assumption is made that this stone pile is capable of absorbing very large compressive forces, but, on the contrary, can resist only feeble tensions (weak mortar will offer little resistance to the work being pulled apart), then an account of masonry action can be given within the framework of modern plastic theory. This theory was devised initially for the design of ductile steel structures, but the powerful new theorems can be ‘translated’ to account for the behaviour of masonry.

Structural considerations
Wren’s magnificent buildings occasionally encountered foundation difficulties, but they seemed to pose no structural problems (or at least there were none, with Hooke to help in the background). The structural considerations for the design of masonry are not those that would be recognised by the modern structural engineer. There are many minor criteria for design, but modern engineers are concerned primarily with problems of strength, stiffness and stability. The structure must be strong enough to carry whatever loads are imposed, including its own weight; it must not deflect unduly; and it must not develop

The central tower at Wells Cathedral was increased in height in 1315–20. The extra weight on the foundations caused uneven settlements of the four crossing piers and the cathedral mason, William Joy, took drastic action in 1338. His curved ‘strainer arches’ are sufficiently massive that they can contain straight thrust lines, and they form (in fact) scissor braces between the four piers.
Building a masonry arch

A simple masonry arch (for example, for a bridge over a river) is made from identical wedge-shaped pieces known as voussoirs. The arch is built on falsework (a temporary supporting framework) since it cannot stand until the last stone, the keystone, is in place. Once complete, the falsework is removed and the arch at once starts to thrust at the river banks. Inevitably the abutments will give way slightly and the arch will spread.

The lower figure, greatly exaggerated, shows how the arch accommodates itself to the increased span. The arch has cracked between voussoirs – there is no strength in these joints, and three hinges have formed. The arch has merely responded in a sensible way to an attack from a hostile environment.

Example: the masonry arch

A simple example makes clear the requirement. A ‘classical’ seventeenth/eighteenth century problem was the analysis of the masonry arch – how does the arch carry its loads? Hooke gave the answer to this problem in 1675: As hangs the flexible chain, so – but inverted – will stand the rigid arch. Thus the ‘catenary’, inverted, will give the line of the resultant forces within the masonry, and this inverted chain must lie within the extrados (upper surface) and intrados (lower surface) of the arch. If, for example, the arch were semicircular (and thus not geometrically similar to the inverted chain), then it would be stable only if it had a certain minimum thickness (about 10% of the radius for a full semicircle).

The ‘safe theorem’

Hooke’s insight is confirmed by the ‘safe theorem’ of the plastic theory of design. The laws of static equilibrium are paramount; the theorem states that if a set of internal forces in a masonry structure can be found that equilibrate the external loads, and which lie everywhere within the masonry, then the structure is safe – safe in the sense that it cannot collapse under those loads.

The importance of this theorem for the analyst is that it is not necessary to determine the ‘actual’ state of the structure; any one satisfactory equilibrium solution demonstrates stability. Indeed, there is no actual state for any structure, whether built of masonry, steel, or any other material. There is, of course, an ideal state imagined by the designer (or by the computer making the design), and there is in practice a state here-and-now, but any small disturbance – a small foundation settlement, a lurch in the wind, an earth tremor, a decay of mortar, a slip in a connexion – will cause a huge alteration to the values of internal stresses. The only real basis for design is to clothe a reasonable equilibrium pattern of forces with suitable constructional material.

Correct shapes and proportions

Thus the design of masonry is transformed into the creation of the correct shapes for individual elements, and for assemblies of such elements into larger units – arcades, cross-vaults, domes. These correct geometrical shapes were established by trial and error, and crystallised into rules; geometry can of course be expressed by numbers – the height of a column must not exceed a certain multiple of its diameter, for example, or again (for Greek temples) that the distance between columns must also depend on their diameter. Numerical rules form the substance of a few chapters in the book of Ezekiel, which preserves part of a builder’s manual of 600 BC; the rules are repeated in Vitruvius, about 30 BC; and they are the basis of a good part of the secrets of the medieval masonic lodges. Structural engineering was design by numbers.

The fifteenth century saw the end of this unbroken tradition of 2000 years. Alberti’s On the art of building of 1452 marks the break – he stressed, above

Stability of an arch

The semicircular arch in Figure (a) below is thick enough to contain easily the line of thrust (the inverted hanging chain, shown as a dashed line) between its upper surface, the extrados, and the lower, the intrados. The arch of Figure (b), however, is only just thick enough, and it is on the point of collapse by the formation of hinges, as shown in Figure (c).
The Gothic cathedral: Westminster Abbey

The diagram shows the cross-section of a typical Gothic cathedral. A stone vault covers the high central space, the nave; the nave is flanked by lower side aisles, themselves vaulted in stone. A tall timber roof covers the nave vault – a stone vault would let in water and need weatherproofing. However, the timber roof is a fire hazard and the stone provides some protection.

The stone vault thrusts horizontally and if there were no side aisles, the massive external buttresses could be placed directly against the north and south walls of the church to discharge the thrusts to the ground. As it is, the vault thrust is taken over the tops of the side aisles by means of flying buttresses. On the north side of Westminster Abbey the lower of the two flying buttresses props the vault; the upper buttress collects any wind force that may act on the timber roof, and also helps to control the tendency of a timber roof to spread.

On the south side of the Abbey, a cloister abuts the wall of the aisle and the main buttressing pier must be placed yet further away. A relatively slender intermediate pier is introduced, and the two upper flying buttresses form two-span inclined arches. The aisle vault also needs propping; this is accomplished by the main buttressing pier on the north side and by a third level of flying buttresses over the cloister on the south.

The cross-section of St Paul’s Cathedral

St Paul’s Cathedral was built over the period 1675–1711 and cost the immense sum of £747,660.

The triple dome of St Paul’s Cathedral (spanning 34 m) consists of a lead-covered timber outer structure, the true conical brick and stone dome supporting the massive lantern, and the inner dome (like the Pantheon, with an ‘eye’) which is all that is seen from the inside of the building.

The essential difference between Wren’s dome for St Paul’s and all previous domes lies in the inclined surfaces of the supporting structure – the masonry never becomes vertical, but follows the line of Hooke’s (three-dimensional) hanging chain. This inclination is disguised externally by the vertical colonnades of the drum and is not detectable internally when standing on the floor of the church; it can however be seen from the Whispering Gallery.
all, the importance of proportion for
correct and beautiful building.
Brunelleschi had made exact
measurements of classical buildings in
Rome; with the invention of printing, an
illustrated Vitruvius could be published,
and the Renaissance of Roman
Architecture was underway.
Two hundred years later, this heritage
was available to Wren. He had no need
to learn proper shapes by cutting stone
in the yard; correct proportions which,
by experience, guaranteed stability,
were to be seen abroad (one year in
Paris) and in books. Only the dome was
difficult – Wren had solved some
geometrical problems with the lath and
plaster dome of St Stephen Walbrook,
but there was real structural engineering
involved with the much larger dome of
St Paul’s. The shape of Hooke’s
catenary gives the reasonable shape for
the two-dimensional arch; Hooke
himself extended his idea to the three-
dimensional dome, and St Paul’s is built
accordingly.

In conclusion
The safe theorem is the rock on which
the whole science of structural
engineering is constructed. The
theorem can be put simply – if the
designer can find a way in which a
structure is comfortable under the
action of certain specified loads, then
the structure is safe. The designer need
find only one such way – this will not be
the way in which the structure chooses
to be comfortable, but if the designer
can find a way, then so can the
structure. Moreover, that way will
change continuously through the life of
the building, but repeated attacks by a
hostile environment will always generate
new states of safe equilibrium (unless,
indeed, the attacks amount to wilful
destruction – violent earthquakes or
acts of war).

Applied to modern building, in steel
or reinforced concrete, for example, the
design, made by laborious hand
calculation or by a computer, refers to
an ideal structure, and the stresses
confidently predicted by the designer
cannot be observed in practice. Never-
theless the elastic solution is one, out of
an infinity of equilibrium solutions, with
which the structure is comfortable.

Further reading
Some of the illustrations are taken from
Chapter 3 of J. Heyman, The science
of structural engineering, Imperial
For a deeper analysis of masonry, see
J. Heyman, The stone skeleton,
Cambridge University Press, 1995
(paperback 1997, 1999).

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